Technical Review

UDC 50 : 669 . 1 . 011

What Outcome is Expected of Mathematics Today? Problem-solving Type Collaboration Focusing on Mathematics

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Abstract

My principle for problem-solving collaboration focusing on mathematics is as follows. 1) Clarification of the principle of the problem by looking at a real-world problem in an abstracted framework using mathematics. 2) Reconstruction of the existing technical concept based on the constructed framework and creating a new technological concept by "think from zero" using mathematics. 3) Applying the technology and attempting to promote it among the manufacturing field and society, and leading them to innovation. Mathematics motivates the manufacturing field and society. I showed the effectiveness of collaboration to solve social problems using examples regarding the heat conduction inverse problems.

Mathematics and Steelmaking Processes

Mathematics exists everywhere in society. Even in a combination of the steelmaking industry and mathematics that appears to be a surprising combination at first glance, mathematics is required in many occasions.

Take the blast furnace shown in **Fig. 1** as an example. A blast furnace is an enormous reactor vessel with a height of approximately 35 m and an inner diameter of approximately 15 m. In that manufacturing process, sintered ores and coke are chemically reacted to remove the molten iron for which the temperature is approximately 1 500°C. Measuring the inside of a blast furnace directly is impossible even when you want to know the inner conditions, so the heat conduction conditions inside the refractories are estimated such that the temperature behavior of the thermocouples buried in the bottom refractories matches the solution of the nonstationary heat conduction equation to the greatest extent possible. A mathematical method called the inverse problem in which the cause is identified from measured results is used here.

When the heat flux flowing from molten iron into the surface of the inner refractory wall is calculated based on the time-series data on the temperature of the refractories at the two points measured with the thermocouples in Fig. 1-2, differences in the variations of the heat flux as shown in Fig. 1-3 are observed although such differences cannot be recognized by only seeing the measured temperature values. For a blast furnace, when the solidified phase that usually protects the refractories was melted and the temperature of the refractories increased, the air supply to the blast furnace is stopped to suspend the production of molten iron for approximately one day. This operation is called shutdown. The measured temperature values in Fig. 1-2 are important data that shows the processes where shutdown operations were repeated and the solidified phase started reforming stably. The broken lines in the graph show the timing at which shutdown operations were carried out.

The combination of shutdown that is an external stimulus given to the system and the magnitude of variations of heat flux is an important index showing the system's stability.[1, 2]

Table 1 compares the engineering technique that was first used in the inverse problem of heat conduction in a blast furnace[3] to a new mathematical technique that was developed in cooperation with mathematicians[4, 5]. As a mathematical model used for the inverse heat conduction problem, the one-dimensional nonstationary heat conduction equation was used. This is because although the bottom of a blast furnace is three-dimensional, the dominant heat transfer is in the one-dimensional direction from molten iron to the back of the bottom refractories. In addition, the intervals between the thermocouples in the circumferential direction of the bottom are not close, so the measuring accuracy does not match the fineness of the mathematical model, which reduces the calculation accuracy of the inverse problem.

Figure 2 is a schematic diagram showing the locations of thermocouples in the refractories of a blast furnace. In the engineering technique, the calculus of variations is used and the initial conditions should have been known as a precondition. However, the entire temperature distribution inside the blast furnace refractories had

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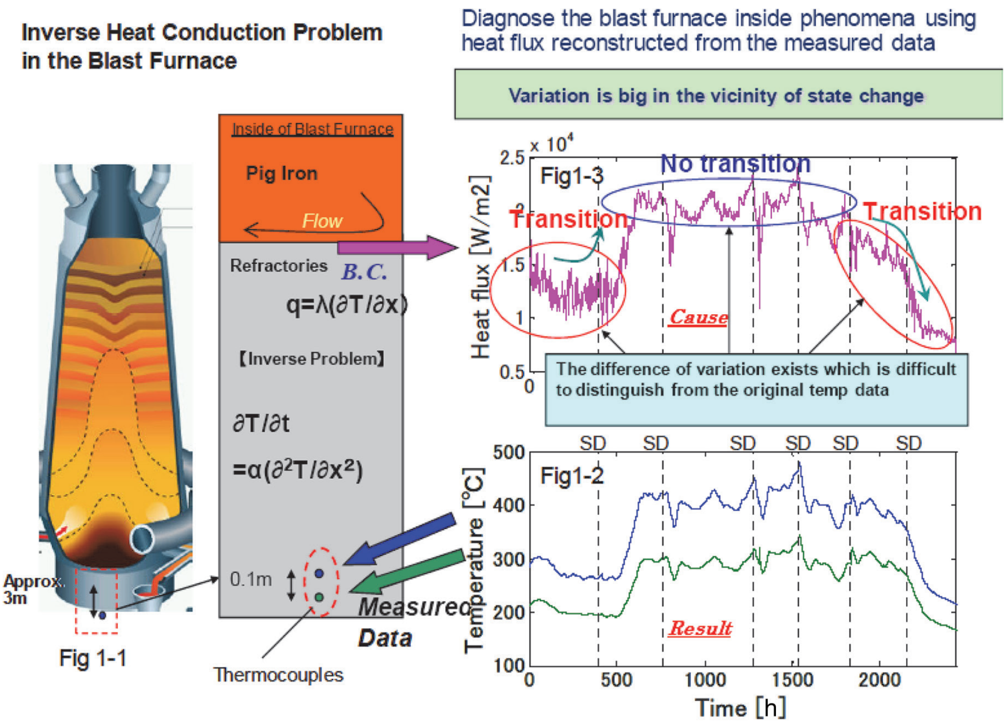

Fig. 1 Application of inverse heat conduction problem to blast furnace

Table 1 Comparison of engineering thinking[3] and mathematical thinking[4] on inverse heat conduction problem in blast furnace

	Engineering Thinking	Mathematical Thinking
$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ (1)	Unsteady heat conduction equation which shows heat balance of refractory inside	Parabolic partial differential equation
$T(x,0) = T_0(x)$ (2)	Initial conditions which show initial temp. distribution of refractory inside	Initial conditions
$\frac{\partial T}{\partial x}(0,t) = f(t)$ (3)	Boundary conditions which show heat conduction through refractory inside wall surface	Neumann boundary conditions
$T(l,t) = \hat{T}_1(t)$ (4)	Boundary conditions which show heat conduction through refractory back wall surface	Dirichlet boundary conditions
$T(l-\Delta x,t) = \hat{T}_2(t)$ (5)	Used for error judgment of variation method	Neumann boundary conditions
	$\frac{\partial T}{\partial x}(l,t) = g(t) \approx (\hat{T}_2(t) - \hat{T}_1(t))/\Delta x$ (6)	

not been measured, so initial temperature distribution was reasonably determined, the time for which the calculation error due to the uncertainty of the set temperature distribution could be ignored due to thermal diffusion effects was judged from experience, and the subsequent calculation results were adopted. In the mathematical technique, mathematical analysis is carried out on the assumption that the initial conditions of the parabolic differential equation and the Neumann boundary condition for one side are unknown and both the Neumann boundary condition and the Dirichlet boundary condition are given as the boundary conditions for the other side by measurement.

Figure 3 shows that the mathematical structure of the cause-and-effect relationship between the two types of measurement data and the two types of unknown variables is clear. However, numeri-

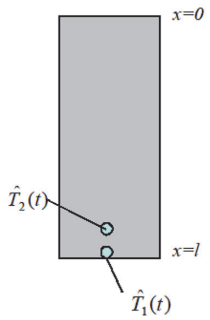

Fig. 2 Thermocouple arrangement and setting of positional coordinates in blast furnace refractories

$$T(x,t)=\sum_{n=0}^{\infty}A_n(x)e^{-\lambda_n t}-\int_0^t G(t-s,x,0)f(s)ds+\int_0^t G(t-s,x,l)g(s)ds$$

measured temperature(B.C.)　I.C.(unknown)　B.C. of inside wall (unknown)　measured heat flux(B.C.)

$$A_0(x)=\frac{1}{l}\int_0^l T_0(y)dy \qquad A_n(x)=\frac{2}{l}\int_0^l T_0(y)\cos\frac{n\pi}{l}y\,dy\cdot\cos\frac{n\pi}{l}x$$

$$\lambda_n=\alpha\frac{n^2\pi^2}{l^2} \qquad G(t,x,y)=\frac{2}{l}\sum_{n=1}^{\infty}e^{-\lambda_n t}\cos\frac{n\pi}{l}\cos\frac{n\pi}{l}y+\frac{1}{l}$$

Fig. 3 Expression of elementary solution of mathematical technique for inverse heat conduction problem (B.C. shows boundary conditions and I.C. shows initial conditions.)

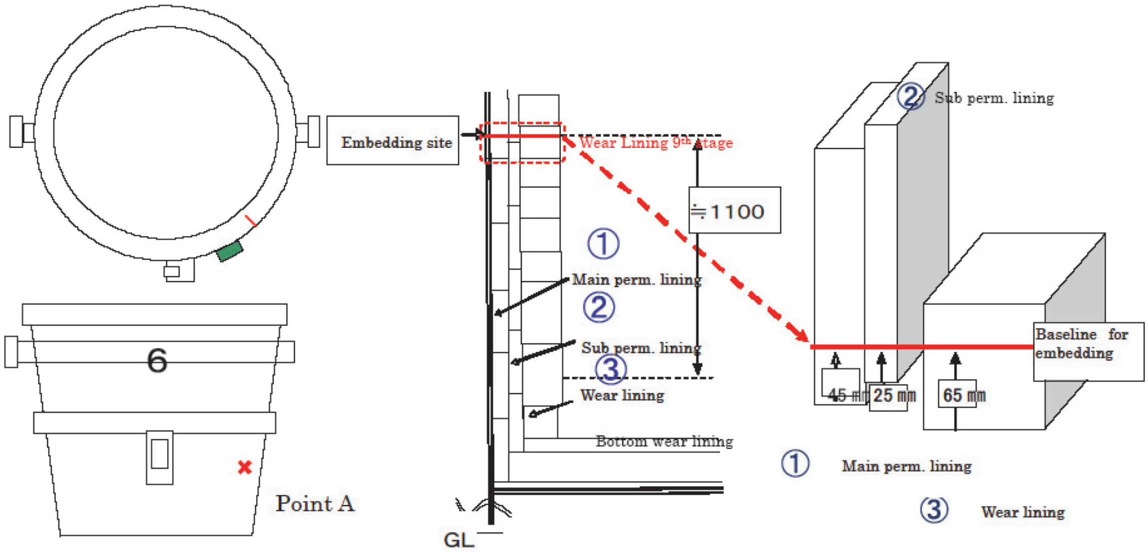

Fig. 4 Steel ladle & thermocouple arrangement outline for adequacy test of inverse problem analysis

cal integral terms ① and ② are very unstable in the numerical calculation of the inverse problem, so the numeral calculation algorithm needs to be devised.[4)]

Using mathematics for this specific problem (inverse problem of heat conduction in a blast furnace) to understand the problem in an abstract framework can show the root of the problem. With this, from the individualized problem (inverse problem of heat conduction in a blast furnace), the engineering concept of the engineering technique can be reestablished as a technique for estimating temperature data inside a material by measuring the temperature and heat flux over time at the same time.

The afore-mentioned engineering concept can be applied to non-destructive diagnosis technologies for equipment materials using infrared thermography. Infrared thermography is a system that measures temperature distribution over the surface of an object using infrared elements and turns the data into images. It has been widely used in various sectors such as industries and medical care, gaining attention recently. **Figure 4** illustrates the outline of a steel ladle. **Figure 5** is an example of the measured temperature of the outer wall of a steel ladle.

Heat flux can be calculated as the sum of radiant heat conduction and natural convection heat conduction from the temperature of the outer wall, so the same mathematical technique used for the blast furnace can be applied without being treated to calculate the internal temperature of the refractories composing a steel ladle. Data on the internal temperature of the equipment material can be used, so the technique may revolutionarily enhance the diagnosability for dissolved loss of refractories, which used to be judged only from data on the temperature of the outer wall. In addition to steel ladles, the exits of various technologies (e.g., abnormality diagnosis of equipment) may be expanded.

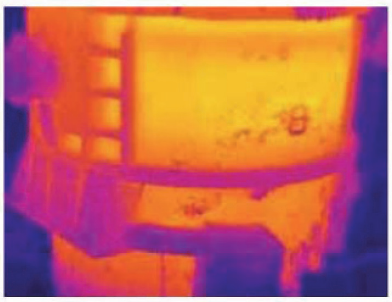

Fig. 5 Example of the outer well temperature of the steel ladle measured by infrared thermography camera

Figure 6 is an example in which multiple thermocouples were experimentally installed inside of the refractories at point A in Fig. 4 to verify the accuracy of the calculation results of the inverse

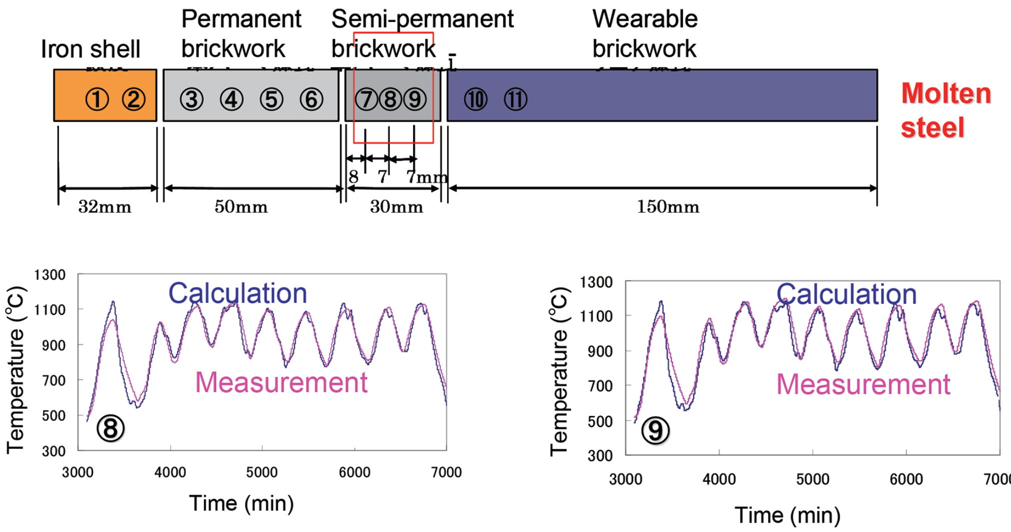

Fig. 6 Example of verification result of mathematical technique for inverse heat conduction problem

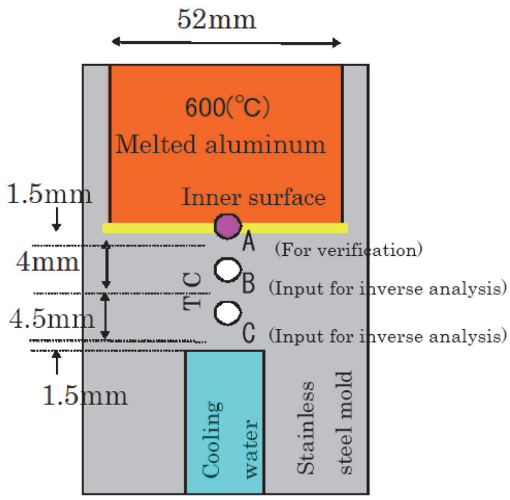

Fig. 7 Outline of laboratory experimental device used to generalize the calculation error of inverse heat conduction problem in engineering manner

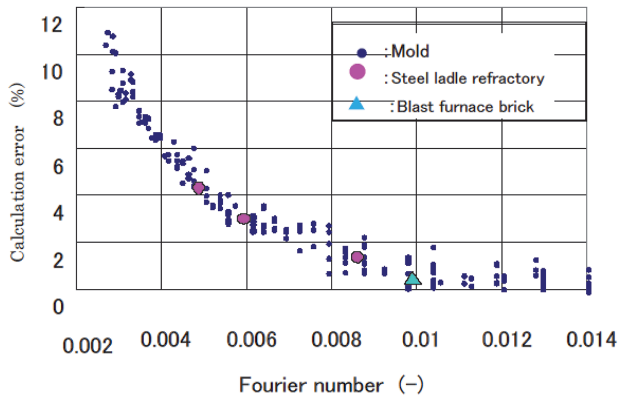

Fig. 8 Generalization of calculation error of inverse heat conduction problem by dimensionless number

problem. The calculation results match the measured values to a level that is satisfied by the manufacturing sections. The accuracy required for a mathematical technique depends on the characteristics of the problem. To evaluate characteristics of the calculation error in the technique for the inverse heat conduction problem described above, a test mold shown in **Fig. 7** was made and an experiment was carried out. **Figure 8** shows the experimental results. Temperatures at point A were calculated based on the data on temperatures measured by the thermocouples installed at points B and C; a value obtained by dividing the difference between an actual value and calculated value by the actual measured temperature was defined as a calculation error; and the result was organized by the Fourier modulus.

The Fourier modulus ($kt/(l^2c\rho)$) is a dimensionless number defined by the ratio of the amount of heat transfer made by heat conduction to the heat amount accumulated in the material. Where, k is the thermal conductivity of the material, t is the time required for temperature change by 1°C, l is the distance between the location at which the temperature was measured and the operation side of the material, c is the material's specific heat, and ρ is the material density. It shows that as the Fourier modulus becomes smaller, the calculation error becomes larger; and it means that as it is more difficult to transmit heat to the thermometer, the reliability of the calculation decreases. The cases of the steel ladle and blast furnace described above are within the variations of the error curve in Fig. 8 and it serves as a guideline for the application limit of the mathematical technique. In addition, this mathematical technique can estimate, for example, whether a technology can be applied even to a completely new system consisting of different materials before the experiment and measurement in detail. This is an example of the generalization of a technology that was made possible by combining a mathematical technique with an engineering mindset—in other words, by collaboration between mathematicians and corporate researchers.[6)]

Roles of Mathematics in Problem-solving Collaboration and Expectations for It

Figure 9 is a form for problem-solving collaboration between mathematicians and corporate researchers that has produced satisfactory results.[5)] To allow individual mathematicians to exhibit their maximum ability, multiple task force teams work in parallel for a

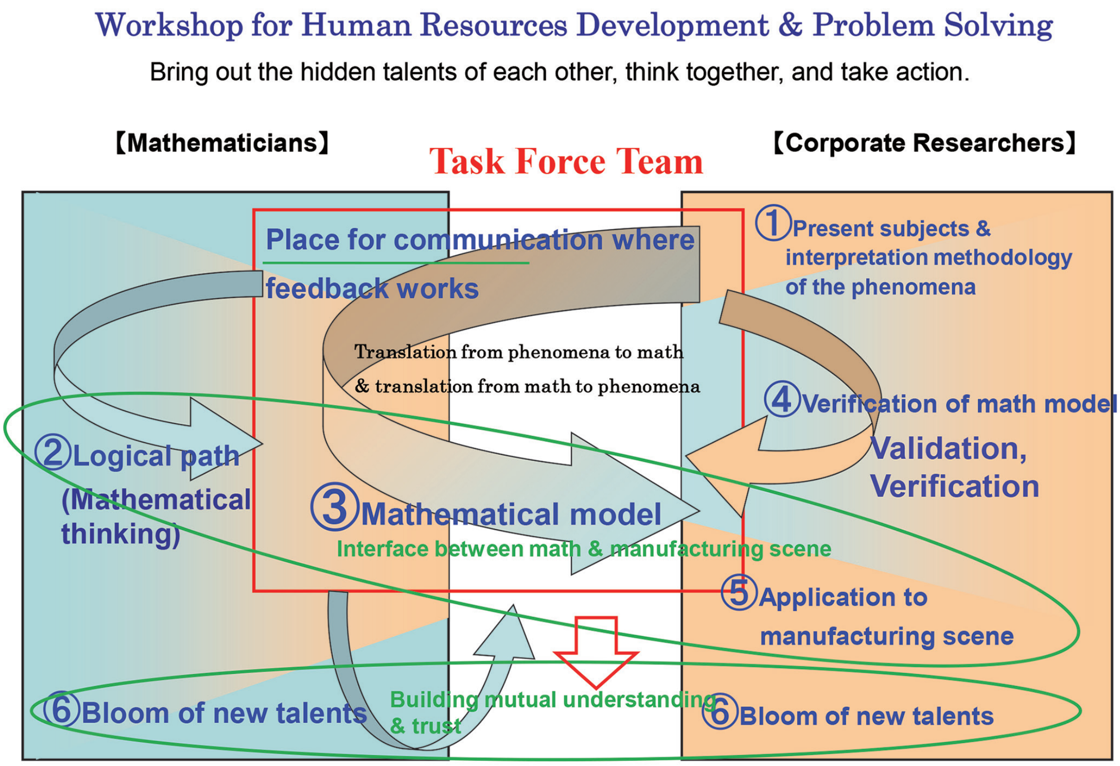

Fig. 9 Style of collaboration between mathematicians and corporate researchers

single problem. Members (mathematicians) are flexibly selected based on the characteristics of each problem.

Such collaboration opportunities work to develop human resources in addition to problem-solving. The center of all items is a place of communication where feedback functions. A mathematical model completes through feedback from various viewpoints such as how to interpret a phenomenon, mathematical mindset, and verification of the appropriateness and accuracy of the mathematical model. Mathematical models serve as interfaces between mathematics and manufacturing sites.

Where, mathematical models for solving problems at companies refer to a series of ways of thinking (procedures) to interpret the actual nature of phenomena occurring at manufacturing sites mathematically. They are not simple applications of existing mathematical techniques, but joint deliverables from discussion between mathematicians and corporate researchers. They work to solve problems at companies and produce new mathematical discoveries.[7, 8] We aim to elicit the abilities of both mathematicians and corporate researchers and develop talented persons who will be hubs between mathematics and society while mathematicians and corporate researchers understand each other more and their relationship of trust is established through the development of mathematical models.

In addition, mathematical models serve as guidelines for understanding phenomena and they work to encourage related parties to recognize problems and share information and to elicit various ideas to solve the problems. We think that sharing models with workers at manufacturing sites who actually handle such phenomena and sophisticating such models in cooperation with them is the best way to find the optimum solution in a limited period of time. For that purpose, a model should be able to capture the nature of the phenomenon and it needs to be easy to understand at the same time. The ability to use mathematics is tested.

To display the advantage of adopting a mathematical mindset—replacement of a specific problem with a universal concept—in the real world at maximum, problem-solving collaboration with mathematics as its core aims at the items listed below.

(1) Capture problems in the real world in frameworks abstracted by mathematics and clarify their roots.
(2) Reestablish existing engineering concepts based on the frameworks formulated by mathematics.
(3) Create exits of technologies and work to spread technologies to manufacturing sites and society to lead to innovation.

The unexpected application of theories that mathematicians have formed from pure mathematical interest, such as CT scan, theory of cryptography, and wavelet, has been realized by researchers other than mathematicians through the passage of time. Stimuli from such application sides allow the sectors in mathematics to develop on a new level. The history up until now has demonstrated that the application of mathematics and mathematical science to other sectors and the development and deepening of theoretical study due to stimuli from such application have occurred in this form almost constantly.[9] Problem-solving collaboration with mathematics as its core has a high potential to reduce time gaps between mathematical theories and achievement of technologies that advance society significantly.[10]

In addition, mathematics is universal, so mathematical theories can be realized without depending on individual phenomenon and data, being able to maintain neutrality. This neutrality is a major advantage of mathematics in that mathematics can create new give and given problem-solving collaboration forms for various sciences, engineering, and industry circles as a source of innovation.[7–9, 11, 12]

This paper is a revised version of the report in “Mathematics Seminar” published in July 2010.

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