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An Analytical Study on Two-Plate-Girders Skew Bridges

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Abstract

Recently, rationally-designed bridges are popular types of steel bridges in Japan because of the reduced construction costs and labor. They comprise long spanned prestressed concrete slabs and steel plate-girder bridges without diagonals and laterals. However, much attention is being paid because their applicable shapes have heretofore been limited in construction. This report describes their structural behavior with skew and bending at piers beyond ordinary bridge design specification through the numerical analysis.

1. Introduction

Because public construction budgets were cut over the last few years, new bridge structures to reduce construction costs have become more widely studied. Among the myriad new structures, many studies reported on rationally-designed bridges by which significant manpower saving can be achieved either in plant fabrication or site construction work. Such structures already account for a number of construction references. A rationally-designed bridge is a long-span steel I girder bridge using prestressed concrete (PC) slabs and having a smaller number of main girders than a conventional bridge, and its structure is much simplified with as small a number of cross beams and laterals as possible. Because of a limited number of construction references, however, the application of the structure is limited by the authorities to bridges having a skew angle of 75 degrees or larger and a radius of curvature of 1,000 m or so[1]. For this reason, to expand the application of the structure, the skew angle and other aspects require further study.

In view of the above, the authors analytically calculated the additional bending stress occurring to slabs on a sample bridge having a skew angle of 60 degrees, a radius of curvature of 800 m or so and main girders forming angles at intermediate supports, and studied methods to reduce stress. The results are reported below.

2. Study Subject Bridge

The bridge selected as the subject of the study is a PC slab, continuous 5-span steel girder, non-composite 2-main-girder bridge having a maximum span of 47.5 m and a PC slab thickness of 310 mm. The outline of the bridge is shown in **Fig. 1** and its main specifications are listed in **Table 1**.

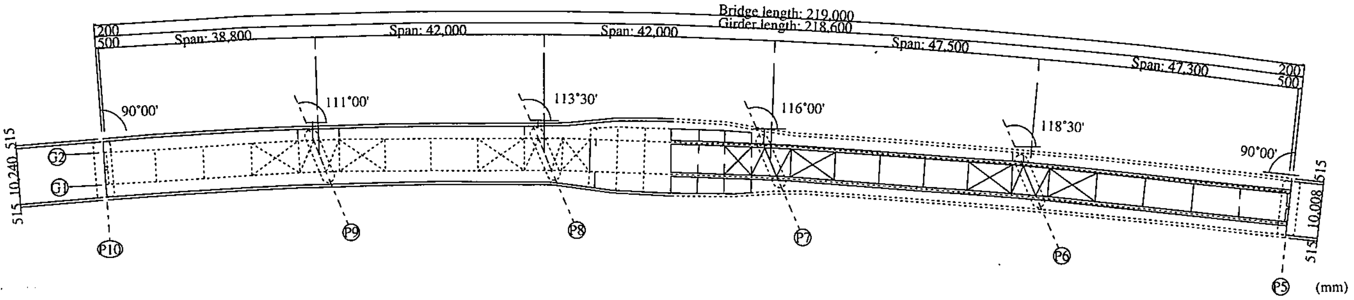

Fig. 1 (a) Plan view of study subject bridge

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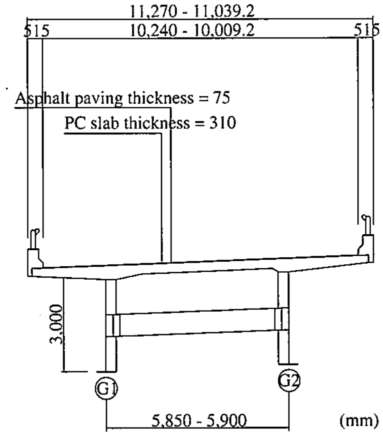


Fig. 1 (b) Section view of study subject bridge

Table 1 Main specification of study subject bridge

Road standard		1st category, 2nd class, standard B
Live load		B
Structure		PC slab, continuous 5-span steel girder, non-composite 2-main-girder bridge
Bridge length		219.0 m
Span		38.8 m + 2 × 42.0 m + 47.5 m + 47.3 m
Effective width		10.009 m to 13.166 m
Horizontal alignment		R = 800 m to A = 450
Longitudinal slope		–2.004% to 3.840%
Skew angle		P10: 90°, P9: 69°, P8: 66.5°, P7:64°, P6:61.5°, P5: 90°
Slab		PC slab (σ_{ck} = 40N/mm^2) Span length of slab = 5.9 m, Slab thickness = 310 mm
Steel materials	Main girder	SS400, SM400, SM490Y, SM570
	Reinforcing bar	SD345
	PC wire	SWPR19-1S21.8 ctc350 (After-bond)

3. Study by Numerical Analysis

The study was done in two stages: first, an in-plane framework analysis to calculate the additional stress resultant caused to the main structure by the skew angle and the out-of-linearity of the main girders at the intermediate supports; then, a 3-dimensional FEM analysis was performed to evaluate the 3-dimensional characteristics of the rationally-designed bridge. Basic models were worked out using the section data calculated according to conventional design specifications based on the results of a grid analysis calculation.

3.1 In-plane Framework Analysis

3.1.1 Models and Cases of Analysis

The models for the in-plane framework analysis are as per **Fig. 2**, and the section data of the members of the models are given in **Table 2**. Pin supports were adopted either in the longitudinal axis of the bridge or in the transverse axis of the bridge. The cases listed in **Table 3** were selected for the analysis for the purpose of evaluating the influences of the skew angle, the rigidity of cross beams and the provision of laterals. A dead weight equivalent to that used in the FEM analysis models (to be explained later) was used in the analysis.

3.1.2 Analysis Results

Table 2 Member data of in-plane framework analysis model

	Sectional area A (cm^2)	Sectional rigidity I_y (m^4)	Torsional rigidity J (m^4)
Main girder	1.64×10^0	3.39×10^{-1}	1.00×10^{-5}
Cross beam	3.30×10^{-2}	1.13×10^{-3}	1.00×10^{-5}
Cross beam on intermediate support	1.60×10^0	1.77×10^{-2}	1.00×10^{-5}
Lateral	6.02×10^{-3}	2.06×10^{-5}	1.00×10^{-5}

Table 3 Cases for analysis

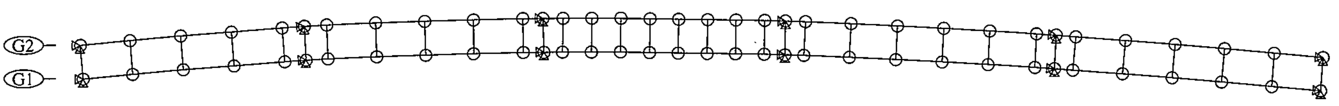
Analysis case	1	2	3	4	5	6
Analysis model	1	2	3	1	2	3
Angled girders	○	○	○	○	○	○
Skew angle	×	○	○	×	○	○
Lateral	×	×	○	×	×	○
Reinforcement of intermediate cross beam with concrete	×	×	×	○	○	○

Note: ○ provided, × not provided. The reinforcement of intermediate cross beams with concrete was applied only to the intermediate cross beams adjacent to the cross beams on intermediate supports. The sectional data of the reinforced intermediate cross beam are the same as the cross beams on intermediate supports.

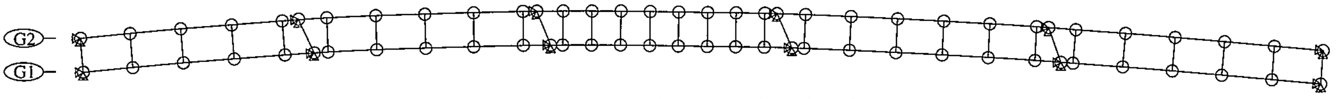
Table 4 Bending moment of cross beam on intermediate support P6

(kN·m)

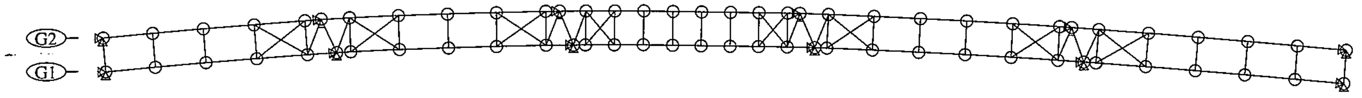
Analysis case	1	2	3	4	5	6
G1 girder side	–399.6	–454.3	–452.5	–399.6	–454.3	–452.5
G2 girder side	401.3	596.9	597.5	401.2	596.9	597.5



(a) Analysis model 1: no skew angle, no laterals



(b) Analysis model 2: with skew angle, no laterals



(c) Analysis model 3: with skew angle and laterals

Fig. 2 In-plane framework analysis models

The resulting stress was the largest at the cross beam on intermediate support P6. The bending moment of the cross beam on intermediate support P6 is shown in **Table 4** for each of the analysis cases. As a result, the following findings were obtained:

(1) The bending moment of the cross beam on intermediate support P6 is influenced significantly by the twist of the entire bridge caused by the skew angle and the out-of-linearity of the main girders.

(2) The bending moment of the cross beam on intermediate support P6 is influenced more by the out-of-linearity of the main girders than the skew angle.

(3) The influences of whether there are laterals and the rigidity of the intermediate cross beam adjacent to the intermediate support over the bending moment of the cross beam on intermediate support P6 are small.

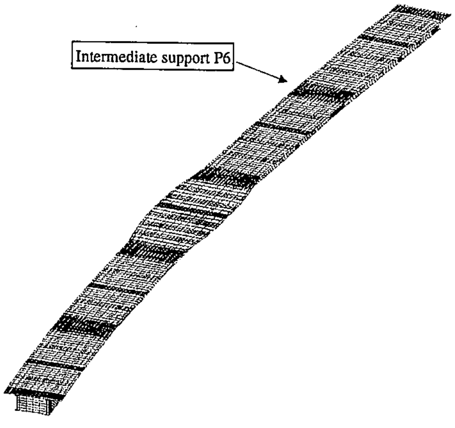


Fig. 3 (a) Overall view of FEM analysis model

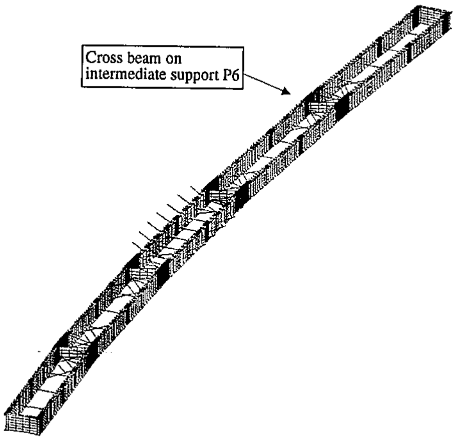


Fig. 3 (b) Main structure of FEM analysis model

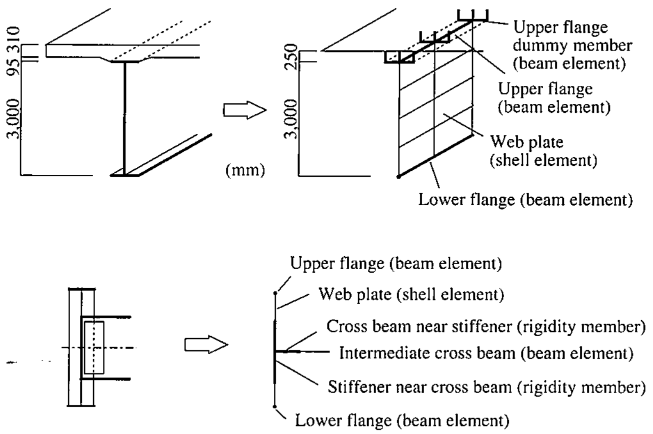


Fig. 3 (c) Modeling of main and cross beams

3.2 3-Dimensional FEM Analysis

3.2.1 Models and Cases of Analysis

The models for the 3-dimensional FEM analysis are shown in **Fig. 3**. In the models, the slabs, the web plates and flanges of the main girders and the cross beams on intermediate supports were counted as shell elements and the other members as beam elements, based on the sectional data of each member shown in **Table 5**. Note that the prestress of the slabs was not taken into account in the analysis. The restriction conditions in the analysis models were as follows: with respect to the movement in the longitudinal axis of the bridge, it was fixed only at P10, and all the other supports were movable; the movement in the transverse axis was restricted at all the supports; and rotational movements were not restricted in any directions.

For the purpose of grasping the additional bending stress occurring to the slabs and studying the method of reducing the stress, the 5 cases shown in **Table 6** were selected. The dead weight shown in **Fig. 4** was imposed on the entire analysis model. In Case 5, the live load shown in **Fig. 5** to maximize the torsion of the cross beam on intermediate support P6 was imposed in an offset pattern.

3.2.2 Analysis Results

(1) Under dead load:

Fig. 6 shows the equivalent stress contour of main girders calculated based on Case 1 and the stress contour in the transverse axis of the bridge at the upper surface of slabs. In Case 1 where only the

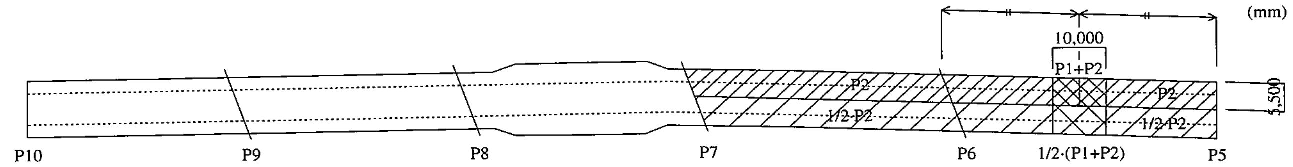
Table 5 Data of 3-dimensional FEM analysis model

Member		Elastic modulus (N/mm^2)	Poisson's ratio
Steel member		2.059×10^5	0.3
Concrete member		3.039×10^4	0.167
Plate element			
Member		Plate thickness (mm)	
Slab		310 (concrete section)	
Main girder	Flange	22 to 48	
	Web plate	16 to18	
Cross beam	End support	800 (concrete section)	
	Intermediate support	800 (concrete section)	
Beam element			
Member		Sectional area (mm^2)	Geometrical moment of inertia (mm^2)
Intermediate cross beam		23.55×10^3	2.01×10^9
Lateral		6.16×10^3	2.08×10^7
Vertical stiffener	Intermediate support	21.12×10^3	1.80×10^8
	Others	6.16×10^3	7.65×10^7

Table 6 Analysis Cases

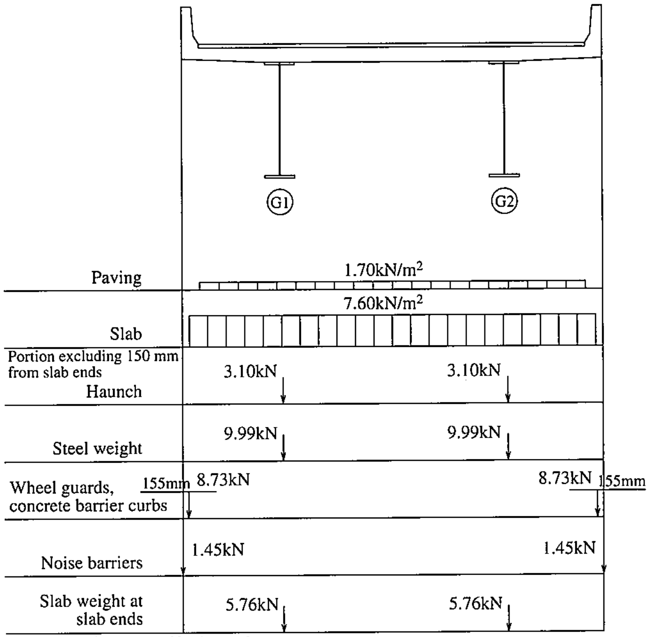
Analysis case	1	2	3	4	5
Live load	D	D	D	D	D + L
Cross beam	I	2·I	I	I	I
Lower lateral	○	○	×	○	○
Upper lateral	×	×	×	○	×

Note: ○ provided, × not provided. D: dead load, L: live load, 2·I: twice the standard lateral rigidity of Case 1



P1 = 10kN/m^2
P2 = 3.5kN/m^2

Fig. 5 Loading of live load



From slab end support to No. 1 cross beam

Fig. 4 Strength under live and dead loads

bridge dead weight was taken into account, the equivalent stress occurring to the main girders was 150 N/mm^2, smaller than the allowable stress of steel materials, 210 N/mm^2. The bending/tensile stress of slab concrete in the transverse axis of the bridge was as small as 2.5 N/mm^2, satisfying the design strength when design prestress was taken into account. The bending/tensile stress applied to a slab, however, tended to concentrate near each support; especially at P6, where both the span and the skew angle were large, the bending/tensile stress in the transverse axis of the bridge was prominent. This trend was the same in all the analysis cases. The stress in the transverse axis of the bridge at the upper face of slabs near P6 and the stress resultant of steel members calculated in the analysis cases are shown in **Tables 7 and 8**, respectively. It was made clear from these results that the stress occurring to the slabs was the same in any of the analysis cases and that there was no large difference in the stress resultant in the laterals and cross beams.

(2) Under offset live load:

Fig. 7 shows the stress contour in the transverse direction occurring to slabs under an offset live load. Although the compressive stress occurring to the slabs at the center of main girders on P6 in Case 5 was approximately 1.5 times that in Case 1, the stress at the other parts was about the same as in Case 1. The stress resultant in the laterals and cross beams was small, too. It was made clear from these results that exceed stress little occurred at any of the bridge sections even under an offset live load.

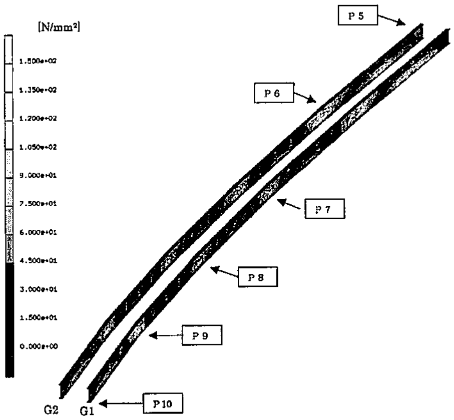


Fig. 6 (a) Equivalent stress contour (main girders)

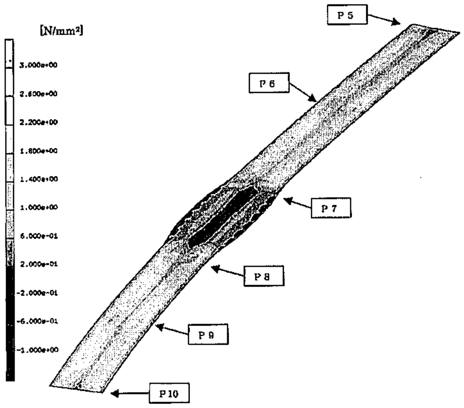


Fig. 6 (b) Stress contour in transverse axis of the bridge (slab upper face)

Table 7 Stress in transverse axis of the bridge near intermediate support P6 (slab upper face) (N/mm^2)

Analysis case		1	2	3	4	5
C23	G1	1.02	1.00	0.99	0.99	1.01
	G2	1.00	1.00	0.97	1.01	1.15
P6	G1	1.90	1.90	1.82	1.90	2.05
	G2	1.65	1.65	1.57	1.66	2.03
C24	G1	1.23	1.22	1.19	1.23	1.27
	G2	0.94	0.92	0.91	0.92	1.05

Note: All the stress figures are in tensile stress.

4. Discussion

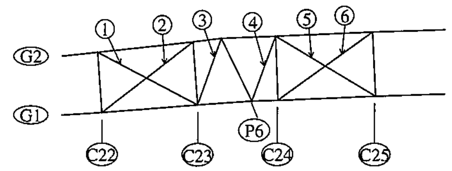
4.1 Slabs

(1) Since the sectional rigidity of slabs is far larger than that of the main structure members such as the main girders, even when

Table 8 Stress resultant of steel members

Analysis case		1	2	3	4
Cross beam on intermediate support	P6	14.7	24.5	–88.2	42.1
Intermediate cross beam	C22	7.8	11.8	–42.1	13.7
	C23	11.8	2.0	–46.1	29.4
	C24	11.8	2.0	–57.8	30.4
	C25	–23.5	–19.6	–44.1	–18.6
Lower lateral	①	3.9	–7.8	—	1.0
	②	–122.5	–124.5	—	–123.5
	③	–56.8	–69.6	—	–54.9
	④	–115.6	–133.3	—	–110.7
	⑤	–94.1	93.1	—	–95.1
	⑥	49.0	34.3	—	45.1
Upper lateral	①	—	—	—	–20.6
	②	—	—	—	–47.0
	③	—	—	—	–66.6
	④	—	—	—	–92.1
	⑤	—	—	—	–43.1
	⑥	—	—	—	–16.7

Note: Positive figures indicate tensile force, and negative figures compression force.



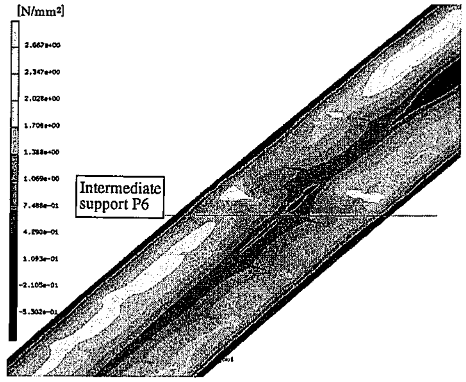


Fig. 7 (a) **Stress contour in transverse axis of the bridge in Case 1 (slab upper face)**

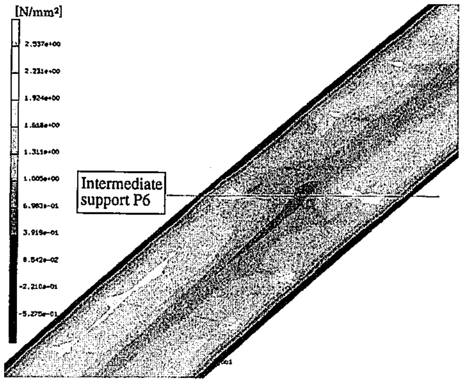


Fig. 7 (b) **Stress contour in transverse axis of the bridge in Case 5 (slab upper face)**

cross beams are reinforced with concrete or more laterals are provided for reducing the stress occurring to the slabs, there will be little difference in the stress occurring to them.

(2) In any of the cases, the bending stress in the transverse axis of the slabs was tensile in the upper surface and compressive in the lower. This is because the total load of wheel guards, concrete barrier curbs and noise barriers installed at the ends of the slabs was considerably large.

(3) No tensile stress occurred to the slabs even in the offset live load model where the additional bending moment on the slabs was the largest. Although a simple comparison with the design stress of the slabs may not be appropriate, it does not seem necessary to especially take into consideration the additional stress in the design of slabs as far as the bridge data assumed in the analysis were concerned.

4.2 Main Structure

(1) Since the sectional rigidity of the slabs of the rationally-designed bridge is large, they bear the most part of dead and live loads. Consequently, when the stress resultant is calculated by grid analysis under the condition that all the loads are borne by the main girders and the cross beams is designed on that basis, more risk than is probable will be accounted for in the design of the steel members and, thus, the redundancy of main girders will be kept.

(2) Since the provision of laterals or the rigidity increase of the cross beam on an intermediate support results in the restriction of the sectional members of the bridge, such a measure may cause an increase in the stress of cross beams and other members.

5. Summary

Reported in this paper were the analytical calculation of the additional bending stress occurring to slabs and the study of the measures to reduce the stress, on a sample bridge having a skew angle of 60 degrees, a radius of curvature of 800 m and main girders forming angles at intermediate supports with those of adjacent spans (the application of the rationally-designed bridge to such a bridge is not allowed presently). As a result, it was confirmed that no stress problem was involved even when the bridge of the studied structure was designed in the same procedure as conventional plate girder bridges and that no concrete reinforcement of cross beams and no laterals ware required specifically. In the case of a rationally-designed bridge having a skew angle at bridge ends or a smaller radius of curvature, however, it is possible that exceed stress resultant occurs near intermediate supports. Thus, further data accumulation through studies is required.

Reference

1) Japan Highway Public Corporation: Design Guideline II: Bridge Design. 1998-7