1. Introduction
An elongation known as the initial stretch occurs when pulling a structure consisting of more than one component, resulting in the structure shown schematically in Fig. 1. In the actual usage environment of bridges, efforts are made to minimize the initial stretch of, for example, their ropes and chains. Conversely, there are many manufacturing processes in which the initial stretch works favorably. In the tire-manufacturing process, each green tire into which a cord has been inserted is first placed into a mold with the prescribed tread pattern and then pressed against the mold to transfer the tread pattern to the tire. In this process, the cord inside the tire rubber is extended. Therefore, cords to be used in tires display an initial stretch of approximately several percent during tire production; however, the initial stretch does not occur after production of tires. In recent years, the manner in which tire cords are stranded has become so diversified and complex that it has become difficult to predict and control the initial stretch of these cords. On this basis, we conducted FEM analysis to assess whether it would be possible to predict the initial stretch of tire cords without actually stranding tire cords.

In the FEM analysis, reference was made to the studies of Fekr et al. and Nono et al. on the analysis of stresses during flexural deformation of cords and the study of Oh et al. on the analysis of cord deformation, strain, and stress in the stranding process.

2. Specimens and Experimental Procedure
Initially, we assigned wire diameter, strand configurations, cord pitches, and strand pitches with reference to the public patent bulletin of Tamada et al. Using a 0.11-mm-diameter steel wire with a stress–strain curve, as shown in Fig. 2, we prepared a specimen of 1 × 4 × 0.11 strands with a pitch of 1.15 mm and subjected it to a tensile test. In addition, a specimen of 5 × 4 × 0.11 a multi-strand cord (Table 1) was prepared and subjected to a tensile test.

Figs. 3 and 4 illustrate examples of the FEM models of the...
multi-strand cord used in the present study. Fig. 3 shows a \(1 \times 4 \times 0.11\) strand model with 1.15 mm strand pitch. This model was used to study cases in which initial stretch occurred. Fig. 4 shows a \(5 \times 4 \times 0.11\) cord model with 1.42 mm strand pitch and 2.8-mm cord pitch. This model was used to verify the tensile test results. Additionally, the models presented in Table 2 were implemented to study the effects of strand and cord pitches. The shape\(^7\) of each strand was designed using homemade software, and FEM calculations were performed by applying MARC to the designed shape.

The calculation conditions used are described below. Each individual strand was constrained at both ends. The position of one end was constrained via displacement control node A, and displacement in the cord axial direction was applied to the other end via displacement control node B. The normal vector of the initial end forms a certain angle with the cord axis; because that angle is considered to decrease when the cord is extended, the end was allowed to rotate freely. The same conditions were set for each of the four strands of the model shown in Fig. 3 and for each of the twenty strands of the model shown in Fig. 4, and the displacement and load at displacement control node B were sampled.

It should be noted that the actual specimens are considered to have been subjected to work hardening and residual stress due to plastic deformation in the stranding process. However, because the purpose of the FEM analysis in the present study was to estimate the initial stretch at low stresses, we conducted an elastic analysis without considering initial strain and initial stress. The material constants used in the calculations were the modulus of elasticity (= 195,000 MPa), obtained from Fig. 2, and Poisson’s ratio (= 0.3). The friction between strands was assumed to be 0.

### 3. Test Results and Analysis

#### 3.1 Test results obtained with \(1 \times 4 \times 0.11\) strand and its analysis

Fig. 5 illustrates the results of a tensile test and analysis of a \(1 \times 4 \times 0.11\) strand. First, FEM calculations were performed using a model with tightly wound strands. However, because the model did not exhibit any initial stretch, another model was created with loosely wound strands. Therefore, there was good agreement between the calculated and measured results. Consequently, it is evident that a gap between strands is required for initial stretch to manifest itself.

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**Table 2** List of results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Strand pitch</th>
<th>Cord pitch</th>
<th>Initial stretch</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.42 mm</td>
<td>3.5 mm</td>
<td>3.0% 3.2%</td>
</tr>
<tr>
<td>B</td>
<td>2.8 mm</td>
<td>5.5%</td>
<td>4.2%</td>
</tr>
<tr>
<td>C</td>
<td>1.15 mm</td>
<td>2.3 mm</td>
<td>4.1% 4.4%</td>
</tr>
</tbody>
</table>

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**Fig. 2** Stress-strain curve of steel wire

**Fig. 3** \(1 \times 4 \times 0.11\) strand model for FEM

**Fig. 4** \(5 \times 4 \times 0.11\) cord model for FEM

**Fig. 5** Load-elongation curve of \(1 \times 4 \times 0.11\) strand
Assuming that the strands were deformed without being constrained by each other during the initial stretch, the model shown in Fig. 3 was broken down, as shown in Fig. 6, and subjected to calculations based on the assumption that it was pulled such that individual strands did not make contact with each other. The calculation results are shown in Fig. 7, which presents an enlarged view of the initially stretched part shown in Fig. 5; “simple springs” shown in the figure are the calculation results obtained with the model shown in Fig. 6. In initial stretch region, it is considered possible to estimate the initial stretch approximately using a simple spring model.

Although we noted that the calculated and measured results agreed well in the above case, the calculated values shown in Fig. 7 are larger than the measured values in the 0.2%-1.0% region. However, the estimated and calculated results almost overlap in the 1.0%-1.8% elastic elongation region, suggesting that there is a transition region between the initial stretch and the elastic elongation in the calculations. We studied the phenomenon occurring in the transition region, thereby finding that the strands were in point contact with each other in the region where the calculated initial stretch was 0%-0.2%; the region in which the calculated initial stretch was 0.2%-1.0%, the number of point contacts increased, and the strands were partly in linear contact with each other as the calculation continued. The strands were in complete contact in the elastic elongation region with an initial stress of 1.0%-1.8%. The above transition region was almost unrecognizable from the measured values. Therefore, we assume that the change from a point contact to an overall contact occurs promptly during the measurement.

FEM calculations in the present study used two-pitch models, which were considerably shorter than the actual specimens used in the tensile test. It is surmised, therefore, that the specimen edge constraining conditions may have influenced the calculation results. Clarifying the effect of pitch length is a task that may be tackled in future.

3.2 Test results obtained with 5 × 4 × 0.11 cord and its analysis

Fig. 8 illustrates the results of the tensile test of 5 × 4 × 0.11 cord specimens. Of the three specimens, specimen C exhibited the largest initial stretch, followed by specimen B and then specimen A, illustrating a tendency for the modulus of elasticity to decrease during elastic elongation.

Fig. 4 shows the 5 × 4 × 0.11 one-pitch cord model with 1.42-mm strand pitch and 2.8-mm cord pitch. In addition, a 5 × 4 × 0.11 one-pitch cord model with 1.42-mm strand pitch and 3.5-mm cord pitch and that with 1.15 mm strand pitch and 2.3 mm cord pitch were prepared and tested. The gap between the strands of these models was assumed to be the same as that in the 1 × 4 × 0.11 strand model.

First, in order to confirm the possibility of reproducing the modulus of elasticity during initial stretch with a simple spring model, each of the above models was broken down into strands and subjected to FEM calculations on the assumption that they were pulled in such a manner that they did not make contact with each other. The calculation results are shown in Fig. 9. Clearly, it is possible to predict the modulus of elasticity of a multi-strand cord during its
initial stretch using a simple spring model even though the measured values are somewhat larger than the calculated values.

Next, the separate strands were wound into multi-strand cords, and their elongations were calculated. The calculation results are shown in Fig. 10: two auxiliary lines can be seen in addition to the lines that indicate the measured and calculated values. The auxiliary line for low elasticity depends on the modulus of elasticity obtained with the simple spring model, whereas the auxiliary line for high elasticity represents the region in which the modulus of elasticity of a multi-strand model becomes almost linear. The point of intersection between the two auxiliary lines is assumed to be the initial stretch obtained by the calculation. The auxiliary line for high elasticity is nearly equal to the gradient of the elastic elongation region of measured values; thus, it is also possible to estimate the modulus of elasticity during elastic elongation.

The results obtained are summarized in Table 2. On the whole, we consider the accuracy of our FEM analysis to be such that the degree of initial stretch can be estimated satisfactorily, although several problems such as the calculated values are somewhat larger than the measured values; there is a marked difference between the calculated and measured values obtained with specimen B; and the transition region from the initial stretch to the elastic elongation is wider than the measured region are yet to be solved.

In the present calculations, in which short-pitch models were used, we assume that the transition region from simple spring to elastic elongation was wider than the measured region under the influence of the constrained ends of the model as in the case of the 1 × 4 × 0.11 strand model. However, simply increasing the pitch length will require considerable calculation time and make the accuracy of calculation unstable. Therefore, it is considered necessary to review the constraining conditions and consider, for example, changing the solver.

4. Conclusions

We conducted an FEM analysis of the initial stretch of multi-strand cords. Our results indicated that the modulus of elasticity of initial stretch is equal to the value obtained when the cord is pulled in such a manner that the strands do not make contact with each other; we also found that the modulus of elasticity of elastic elongation is equal to the value obtained when the strands make complete contact with each other with no gap between them. In addition, we found that it was possible to estimate the amount of initial stretch as the point of intersection between two auxiliary lines obtained from a simple spring model (the modulus of elasticity of initial stretch) and a multi-strand cord model (modulus of elasticity of elastic elongation). Conversely, several problems were found in the present study; these issues must be addressed in future.

References

2) Kawai, T.: Special Steels. 45 (12), 10-13 (1996.12)